A Matching Model with Endogenous Skill Requirements

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Introduction

In this paper, we construct a model that highlights the role of skill in the labor market. "Skill" has been used to explain wage inequality in the United States and the United Kingdom and high unemployment in Europe. For example, Katz and Murphy (1992), who document that wage inequality has increased both within and across skill groups in the United States over the past 25 years, argue that shifts demand in favor of more highly skilled workers are in large part responsible for these trends. Similarly, Krugman (1994) argues that skill-biased technical change has shifted demand in favor of high-skill workers and that this has lead to increased earnings inequality in both the U.S. and the U.K. and to increased unemployment in continental Europe. Woods (1994) makes a similar argument, although he argues that increased trade with the Third World, as opposed to technological factors, lies behind the relative demand shift. In the context of this demand-shift argument, the different developments in the U.S and U.K. versus the rest to Europe are ascribed to differences in policy and labor market institutions (Mortensen and Pissarides 1998). On the supply side, Nickell and Bell (1996), who disagree with the Krugman/Woods explanation of European unemployment, argue that there is less wage inequality in, for example, Germany than in the U.S. and the U.K. because the German skill distribution is considerably more compressed. In Germany, in contrast to the U.S. and the U.K, they argue "we do not find a large segment of the workforce who simply cannot cope with the demands placed upon them by technical change." (p. 306)

In our model, we consider a labor market in which some workers are low-skill, while the remainder are high-skill. The market is one in which employers create vacancies, which can require either a low skill or a high skill. A low-skill vacancy can be filled by either type of worker, while a high-skill vacancy can only be filled by a high-skill worker. Thus, for example, a Ph.D. in nuclear engineering can do rocket science or she can flip hamburgers. A high-school dropout can't do rocket science; he can only flip hamburgers. By our definition, the Ph.D. is a "high-skill worker," and rocket science is a "high-skill job." This definition of skill, which is essentially the one used in Vroman (1987), makes it possible to talk both about low- and high-skill workers and about low- and high-skill jobs. It is intended to capture the fact that many jobs have minimum skill requirements associated with them. Workers with a skill level above the minimum required for a job can be hired, but they may not produce more on that job than a worker with less skill. Note, however, that ours is not the only possible way to think about skill. In particular, one could define skill in terms of efficiency units of labor. Thus, it is often said that a worker is highly skilled if she can complete a job relatively quickly; equivalently, a group of workers is called high-skill if relatively few are required to complete a particular task. This efficiency units notion of skill, while useful for many purposes, is not a natural one for distinguishing between low- and high-skill jobs.

The labor market is modeled using a matching framework in the spirit of Diamond (1982), Mortensen (1982), and Pissarides (1990) coupled with a Nash bargaining approach to wage-setting. Our matching process is undirected in the sense that a low-skill worker encounters a high-skill vacancy (and thus is unable to consummate the match) with a probability per unit of time that is proportional to the fraction of vacancies that are high-skill. Similarly, a high-skill worker encounters a low-skill vacancy (which is less suited to her talents) with a probability per unit of time that is proportional to the fraction of vacancies that are low-skill. We use this undirected matching process (as opposed to a process in which the low- and high-skill markets are segmented) to capture the idea that, given overall labor market conditions, low-skill workers are better off the greater the fraction of vacancies that are low-skill, and vice versa for high-skill workers. Similarly, all else equal, a firm with a low-skill vacancy is better off the greater the fraction among job seekers that is low-skill. This also allows us to examine the possibility of high-skill workers "crowding out" low-skill workers from the low-skill vacancies when high-skill vacancies are scarce.

We use our model to analyze several phenomena. First, we generate wage dispersion within and across skill groups. In our model, high-skill workers are paid more on average than low-skill workers are; at the same time, high-skill workers in high-skill jobs are paid more than are similar workers who are employed in low-skill jobs. Second, we generate differences in unemployment duration across skill groups. If it is worthwhile for high-skill workers to take low-skill jobs, then unemployment duration among low-skill workers will be higher on average than among the highly skilled. If, on the other hand, high-skill workers don’t work at low-skill jobs, then this pattern can be reversed. Third, we examine how vacancy durations vary across job types. Fourth, as noted above, we analyze “crowding out”, i.e., the situation in which high-skill workers take low-skill jobs. As van den Berg, et. al. (1998) show, this can have deleterious implications for the wages and unemployment experience of low-skill workers. Fifth, we examine the comparative statics effects of (i) increasing the value of maintaining high-skill jobs (interpreted in the context of our model as skill-biased technical change), (ii) decreasing the price of the low-skill output (interpreted as the result of cheap import substitution), and (iii) changes in the underlying skill distribution in the worker population. Sixth, and finally, we examine the effects of a search subsidy (unemployment compensation) on the labor market equilibrium.

Three recent papers are closely related to ours. These are Acemoglu (1998), van den Berg (1998), and Saint-Paul (1996). Acemoglu (1998) is a matching model in which there are “good jobs” and “bad jobs.” In his model, good jobs cost more to set up than bad jobs do, but they are also more productive once filled. The main points of his paper are that in laissez faire equilibrium, not enough good jobs are created and that policy interventions such as unemployment insurance and a minimum wage can enhance welfare by inducing workers to hold out for higher wages, which in turn makes it more attractive to create good jobs. However, in contrast to our model, workers in Acemoglu (1998) are homogeneous; so, by definition, there can be no interaction between “good workers” and “bad workers.” van den Berg (1998) also constructs a model with good jobs and bad jobs. His model is one with a two-point distribution of productivities across potentially active firms. Firms post wage offers, and a continuous distribution of wages is supported in equilibrium by on-the-job search, as in Burdett and Mortensen (1998). The main point of van den Berg’s paper is that models of this type typically have multiple equilibria and that policy interventions (especially a minimum wage) can move the economy from a bad equilibrium (i.e., one in which both firm types operate) to a good equilibrium (i.e., one in which only the high-productivity firms operate). van den Berg’s model is also one in which there is no distinction between good workers and bad workers, although he does consider an extension in which workers differ with respect to the opportunity cost of employment, as in Albrecht and Axell (1984). Finally, Saint-Paul (1996) is a model in which there are both low- and high-skill workers and low- and high-skill jobs. As in our model, firms make a decision about what type of vacancy to open, but, unlike in our model, once a vacancy is posted, there is no interaction between the worker types. That is, the low- and high-skill markets are segmented in the sense that low-skill workers search only where there are low-skill jobs, and similarly for high-skill workers.

The Model

Basic Assumptions

The distribution of skills across workers is exogenous. Specifically, we assume a two-point distribution: a fraction p of the workers in the population has the low skill level, s_1, and a fraction $1 - p$ has the high skill level, s_2. The measure of workers is normalized to one.

Jobs are either vacant or filled. Filled jobs break up at the exogenous rate δ. A job is described by its skill requirement, i.e., by the minimum skill required of a worker hired in the job. The technology is such that when a job is filled the output produced, $x(s, y)$, is given by

$$x(s,y) = \begin{cases} y & \text{if} \quad s \geqslant y \\ 0 & \text{if} \quad s < y \end{cases},$$

where s is the skill level of the worker in the job and y is the job's skill requirement. When a job is filled, its cost is $w(s,y) + c$, i.e., the wage paid to the worker plus a fixed cost. If the job is vacant, the fixed cost must still be paid so that the instantaneous cost of a vacancy is c. When a vacant job is created, its skill requirement is chosen to maximize the value of the vacancy. Given the technology and the distribution of skills across workers, the only skill requirements that will be chosen in equilibrium are $y_1 = s_1$ and $y_2 = s_2$.

Unemployed workers and vacancies are assumed to meet each other randomly according to a matching function $m(u,v)$. We make the standard assumptions about the matching function, including that it is characterized by constant returns to scale so that

$$m(u,v) = m(1, \frac{v}{u})u = m(\theta)u, \text{ where } \theta = \frac{v}{u}.$$

The arrival rate for workers is thus $m(\theta)$. Low-skill workers meet vacancies at the same rate as high-skill workers do, but they do not qualify for the high-skill vacancies. Let ϕ denote the fraction of vacancies that are low-skill; accordingly, the effective arrival rate of employment opportunities for low-skill workers is $\phi m(\theta)$. Similarly, vacancies meet unemployed workers at the rate $\frac{m(\theta)}{\theta}$. All vacancies face the same arrival rate, but vacancies with the high skill requirement meet some workers who are not qualified. Let γ denote the fraction of the unemployed who are low-skill; accordingly, the effective arrival rate to high-skill vacancies is $(1-\gamma)\frac{m(\theta)}{\theta}$. Of course, $\gamma \neq p$ since, in general, low- and high-skill workers will find jobs at different rates.

Match Formation and Wages

We assume that matches are consummated between unemployed workers and vacancies whenever the joint surplus that would be realized by the match is nonnegative. In deriving the conditions under which a match will be formed, we use the following notation: $U(s)$ is the value of unemployment for a worker of type s, $N(s,y)$ is the value of employment for a worker of type s on a job of type y, $V(y)$ is the value of a vacancy of type y, and $J(s,y)$ is the value to the employer of filling a job of type y with a worker of type s. A match will be formed if and only if it generates a nonnegative surplus, i.e., if and only if

$$N(s,y) + J(s,y) \geqslant U(s) + V(y). \qquad \#$$

When a match is formed, the wage, $w(s,y)$, is given by the Nash bargaining condition,

$$N(s,y) - U(s) = \beta[N(s,y) + J(s,y) - U(s) - V(y)] \qquad \#$$

where β is the exogenously given worker's share of the surplus.

We now develop expressions for the various value functions. In doing this, we let r denote the discount rate, which is assumed to be the same for both individuals and firms, δ the exogenous job dissolution rate, and b the instantaneous value of leisure (or the unemployment benefit). We assume that workers live forever and are risk neutral.

We begin with the value of employment for a worker of type s on a job requiring skill y:

$$N(s,y) = \frac{w(s,y) + \delta U(s)}{r+\delta} \quad \text{(conditional on } s \geqslant y\text{)}. \qquad \#$$

This expression follows in the usual way from

$$rN(s,y) = w(s,y) + \delta[U(s) - N(s,y)];$$

that is, the flow value for a worker of type s who is employed in a job with skill requirement y equals the sum of the flow return, $w(s,y)$, plus the expected instantaneous capital loss, $\delta[U(s) - N(s,y)]$.

Similarly, the value to a firm of having a job with skill requirement y filled by a worker of type s is

$$J(s,y) = \frac{y - w(s,y) - c + \delta V(y)}{r + \delta} \quad \text{(conditional on } s \geqslant y\text{).} \qquad \#$$

The values of unemployment for low- and high-skill workers are implicitly defined by

$$rU(s_1) = b + m(\theta)\phi[N(s_1,s_1) - U(s_1)] \qquad \#$$

and

$$rU(s_2) = b + m(\theta)\{\phi \max[N(s_2,s_1) - U(s_2),0] + (1-\phi)[N(s_2,s_2) - U(s_2)]\}. \qquad \#$$

The value of unemployment for low-skill workers incorporates the assumption that low-skill workers cannot do high-skill jobs; thus, the arrival rate of jobs that these workers can do is $m(\theta)\phi$. Similarly, the value of unemployment for high-skill workers incorporates the assumption that these workers are capable of undertaking either low-skill or high-skill jobs. The former arrive at rate $m(\theta)\phi$, and the latter arrive at rate $m(\theta)(1-\phi)$. It may not be worthwhile for high-skill workers to be employed on low-skill jobs; this is why the value associated with the arrival of a low-skill vacancy is $\max[N(s_2,s_1) - U(s_2),0]$. footnote

Finally, the values of low- and high-skill vacancies are

$$V(s_1) = -c + \frac{m(\theta)}{\theta}\{\gamma[J(s_1,s_1) - V(s_1)] + (1-\gamma)[\max(J(s_2,s_1),0) - V(s_1)]\} \qquad \#$$

and

$$V(s_2) = -c + \frac{m(\theta)}{\theta}(1-\gamma)[J(s_2,s_2) - V(s_2)], \qquad \#$$

respectively. The value of a low-skill vacancy reflects the assumption that while both worker types are capable of doing the low-skill job, it may not be worthwhile for high-skill workers to take these jobs; i.e., the value of meeting a high-skill unemployed worker is $\max[J(s_2,s_1),0]$. Similarly, the expression for the value of a high-skill vacancy reflects the assumption that only high-skill workers are able to perform these jobs; thus, the effective arrival rate to high-skill vacancies is $\frac{m(\theta)}{\theta}(1-\gamma)$. Since this is a long-run model with free entry and exit, the values of both vacancy types must be zero; i.e., $V(s_1) = V(s_2) = 0$.

Returning to inequality (1) and substituting, a match will be formed if and only if

$$y - c \geqslant rU(s) \quad \text{(conditional on } s \geqslant y\text{).}$$

Similarly, from equation (2), the wage of a worker of type s on a job requiring skill y can be expressed as

$$w(s,y) = \beta[y-c] + (1-\beta)rU(s) \quad \text{(conditional on } y - c \geqslant rU(s)\text{).}$$

The wage is thus a weighted average of the net output of the match (output minus the fixed cost) and the worker's flow value of unemployment. At most three wages are paid in equilibrium, namely,

$$w(s_1,s_1) = \beta(s_1 - c) + (1-\beta)rU(s_1)$$
$$w(s_2,s_1) = \beta(s_1 - c) + (1-\beta)rU(s_2)$$
$$w(s_2,s_2) = \beta(s_2 - c) + (1-\beta)rU(s_2).$$

High-skill workers are always paid more than low-skill workers. If high-skill workers take low-skill jobs, i.e., if the expression for $w(s_2,s_1)$ is relevant, then they receive a higher wage on these jobs than low-skill workers do. This is because high-skill workers have a more valuable outside option than low-skill workers do; i.e., $rU(s_2) > rU(s_1)$. High-skill workers, however, earn a lower wage on low-skill jobs than they do on high-skill jobs. This simply reflects the different productivities on the two types of job. Finally, note that equation (4), together with the equilibrium conditions, $V(s_1) = V(s_2) = 0,$ imply that

$$J(s,y) = \frac{(1-\beta)[y - c - rU(s)]}{r+\delta}. \qquad \#$$

The Model with Crowding Out

Equilibrium

The nature of equilibrium depends on the parameters of the model. There are two cases to consider. The first is an equilibrium in which it is worthwhile for high-skill workers to take low-skill jobs; the second is an equilibrium in which there is *ex post* market segmentation in the sense that it is not worthwhile for high-skill workers to take low-skill jobs. We refer to the former case as "equilibrium with crowding" since the presence of high-skill workers in low-skill jobs impinges upon the opportunities open to low-skill workers. Since the crowding case has not been treated before in the literature, we analyze it first.

Equilibrium requires that $V(s_1) = 0$ and $V(s_2) = 0$. It is more convenient in this case to work with the equivalent conditions: $V(s_1) = V(s_2)$ and $V(s_2) = 0$. From equations (7), (8), and (9), these conditions are

$$\gamma(\frac{(1-\beta)[s_1 - c - rU(s_1)]}{r+\delta}) - (1-\gamma)(\frac{1-\beta}{r+\delta})(s_2 - s_1) = 0 \qquad \#$$

and

$$-c + \frac{m(\theta)}{\theta}(1-\gamma)(\frac{(1-\beta)[s_2 - c - rU(s_2)]}{r+\delta}) = 0. \qquad \#$$

The derivation of equation (10) (i.e., the condition that $V(s_1) - V(s_2) = 0$) uses the assumption that it is worthwhile for high-skill workers to take low-skill jobs (i.e., that $J(s_2,s_1) > 0$).

Solving for the two unemployment values (again, using $N(s_2,s_1) > U(s_2)$) gives

$$rU(s_1) = \frac{b(r+\delta) + m(\theta)\phi\beta(s_1 - c)}{r+\delta+m(\theta)\phi\beta} \qquad \#$$

and

$$rU(s_2) = \frac{b(r+\delta) + \beta m(\theta)[\phi s_1 + (1-\phi)s_2 - c]}{[r+\delta+\beta m(\theta)]}. \qquad \#$$

Note that $rU(s_1)$ is increasing in both θ and $\phi,$ while $rU(s_2)$ is increasing in θ but decreasing in ϕ.

Holding the skill mix of vacancies constant, both worker types are better off as the ratio of vacancies to unemployment increases. Holding overall labor market conditions constant, low-skill workers are better off the greater the fraction of vacant jobs that are low-skill and high-skill workers are better off the smaller is this fraction.

Substituting the expressions for $rU(s_1)$ and $rU(s_2)$ into equations (10) and (11) gives two equations in three unknowns, namely, $\theta, \gamma,$ and ϕ. To complete the system, we use the steady-state conditions that the flows of low-skill workers into and out of unemployment be equal and that the corresponding flows of high-skill workers also be equal. These two steady-state conditions also allow us to solve for the unemployment rate, u.

The condition that the flow of low-skill workers out of unemployment equals the flow of low-skill workers back into unemployment is

$$\phi m(\theta)\gamma u = \delta(p - \gamma u). \qquad \#$$

Since $\phi m(\theta)$ is the arrival rate of vacancies with low skill requirements and γ is the fraction of the unemployed who are low skill, the flow of low-skill workers out of unemployment is $\phi m(\theta)\gamma u$. The corresponding flow into unemployment is δ times the measure of employed low-skilled workers. This latter measure is $p - \gamma u,$ the measure of low-skilled workers in the population (recall that we normalized the total measure of workers to 1) minus the measure of these workers who are unemployed.

Similarly, the condition that the flow of high-skill workers out of unemployment equals the flow of high-skill workers back into unemployment is

$$m(\theta)(1-\gamma)u = \delta(1 - p - (1-\gamma)u). \qquad \#$$

High-skill workers face an arrival rate of vacancies of $m(\theta),$ and the measure of high-skill unemployed is $(1-\gamma)u;$ thus the flow of high-skill workers out of unemployment is $m(\theta)(1-\gamma)u$. There are $1-p$ high-skill workers, of whom $(1-\gamma)u$ are unemployed. The flow of high-skill workers into unemployment is thus $\delta(1-p-(1-\gamma)u)$.

Equations (14) and (15) can be used to solve for ϕ and u as functions of θ and $\gamma;$ namely,

$$\phi = \frac{(1-\gamma)p(\delta + m(\theta)) - \gamma(1-p)\delta}{m(\theta)\gamma(1-p)} \qquad \#$$

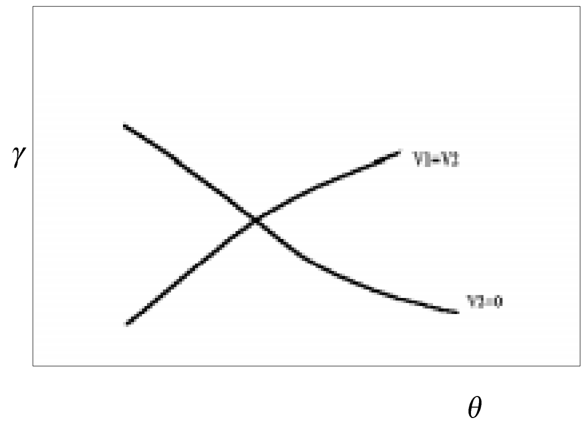
and

$$u = \frac{\delta(1-p)}{(1-\gamma)(\delta + m(\theta))}. \qquad \#$$

Note that ϕ is decreasing in γ and increasing in $\theta,$ the latter so long as $\gamma > p$. This condition in turn follows from the requirement that ϕ be a fraction or equivalently that $1 - \phi = \dfrac{(\gamma - p)(\delta + m(\theta))}{m(\theta)\gamma(1-p)} > 0.$

Returning to equations (10) and (11), we can now see the graphical intuition for equilibrium with crowding. The locus of points such that $V(s_1) - V(s_2) = 0$ (the “equal-value condition”) is upward sloping in the (θ, γ) plane. To see this, note that the left-hand side of (10) is increasing in $\gamma,$ both directly and because $rU(s_1)$ is increasing in $\phi,$ which is in turn decreasing in γ. At the same time, $rU(s_1)$ is increasing in $\theta;$ hence the left-hand side of (10) is decreasing in θ. A similar argument shows that the locus of points such that $V(s_2) = 0$ is downward sloping in the (θ, γ) plane. The intersection of the two curves gives the equilibrium, as illustrated in Figure 1.

Figure 1



More formally, we can solve for equilibrium as follows. The equal-value condition is

$$\gamma[s_1 - c - rU(s_1)] \;= (1-\gamma)(s_2 - s_1).$$

Using equation (12),

$$s_1 - c - rU(s_1) = \frac{(s_1 - c - b)(r+\delta)}{r + \delta + \beta m(\theta)\phi},$$

so this condition can be written as

$$(s_1 - c - b)(r+\delta) = (1-\gamma)[(s_2 - c - b)(r+\delta) + \beta m(\theta)\phi(s_2 - s_1)]. \qquad \#$$

The condition that $V(s_2) = 0$, i.e., equation (11), is

$$c = \frac{m(\theta)}{\theta}(1-\gamma)\left(\frac{(1-\beta)[s_2 - c - rU(s_2)]}{r+\delta}\right),$$

where, from equation (13),

$$s_2 - c - rU(s_2) = \frac{(s_2 - c - b)(r+\delta) + \beta m(\theta)\phi(s_2 - s_1)}{r + \delta + \beta m(\theta)}.$$

Using equation (18), we thus have

$$c = \frac{m(\theta)}{\theta}\,\frac{(1-\beta)}{(r+\delta)}\,\frac{(s_1 - c - b)(r+\delta)}{r + \delta + \beta m(\theta)};$$

that is,

$$c(r + \delta + \beta m(\theta)) \;= \frac{m(\theta)}{\theta}(1-\beta)(s_1 - c - b). \qquad \#$$

Given standard assumptions on $m(\theta)$, equation (19) has a unique solution for θ. If we insert this solution into the equal-value condition or into $V(s_2) = 0$, recognizing that $\phi = \phi(\theta, \gamma)$, we then get a quadratic in γ. This quadratic will have a unique solution consistent with $1 > \gamma > p$.

In the next subsection, we illustrate the properties of equilibrium with crowding by solving the model for a specific functional form for $m(\theta)$ and for specific parameter values. Before doing this, however, we emphasize that the model solution explained above was derived conditional on the assumption that it was worthwhile for high-skill workers to match with low-skill vacancies. That is, we were implicitly assuming that $s_1 - c \geq rU(s_2)$. When we compute the equilibrium, we must, of course, check to see that this condition is satisfied.

Comparative Statics

To illustrate the equilibrium with crowding out, we now solve the model, assuming a specific functional form for $m(\theta)$ and values for the exogenous parameters. Given a functional form for $m(\theta),$ the solution for θ follows directly from equation (19), and once we have $\theta,$ the solution for γ follows from the equal-value condition. Finally, given θ and $\gamma,$ we solve for ϕ and μ using equations (16) and (17). We emphasize that our solution is an analytical one; i.e., it is not necessary to solve the model numerically. Further, the model can be solved analytically for any functional form $m(\theta)$ and for any parameter configuration such that equation (19) has a unique solution for θ. Of course, we need to ensure that the solution is reasonable and that it is, as assumed, worthwhile for high-skill workers to take low-skill jobs.

We also examine how our solution varies with changes in selected parameters. Specifically, we examine the comparative statics of varying (i) $b,$ the unemployment compensation parameter, (ii) $s_2,$ the output produced in the high-skill job, (iii) $p,$ the fraction of the workforce that is low-skill, and (iv) $\beta,$ the workers' share of the surplus.

We use the matching function $m(\theta) = 2\sqrt{\theta}$, and in our baseline case we assume that $s_1 = 1,$ $s_2 = 1.1,$ $p = .5,$ $b = 0,$ $\beta = .5,$ $\delta = .3,$ $c = .1,$ and $r = .05$. This matching function and these parameter values were chosen to generate an unemployment rate close to the one currently prevailing in the United States. The first row of Table 1 presents the solution for the baseline case, rows 2-4 of the table examine the effects of increasing $b,$ and rows 5-7 of the table show the effects of increasing both b and s_2.

The most important feature of our baseline case is that high-skill jobs are only ten percent more productive than low-skill jobs. This leads to an equilibrium in which virtually all vacancies are low-skill ($\phi = .995$). Since there are very few high-skill vacancies, the rate at which high-skill workers find jobs is only slightly above the corresponding rate for low-skill workers. This leads to an outcome in which low-skill workers are only slightly overrepresented in the pool of unemployed ($\gamma = .501$). This further implies that high-skill workers, when they are employed, are almost always working at low-skill jobs. Low-skill and high-skill workers' unemployment rates are almost identical, and when they are employed, the two worker types are almost equally productive. As a result, the unemployment values for low-skill and high-skill workers are almost identical. This explains why $w_{11},$ the wage that a low-skill worker receives on a low-skill job, and $w_{12},$ the wage that a high-skill worker receives on a low-skill job, are essentially the same. Of course, those few fortunate high-skill workers who find one of the rare high-skill jobs earn substantially more than they would have earned on a low-skill job. For the high-skill worker, the wage premium associated with the high-skill job, i.e., $w_{22} - w_{21},$ is precisely one half of the productivity increment, i.e., $s_2 - s_1$. This follows from our assumption that $\beta = 0.5$.

Our baseline case generates an unemployment rate of 5%, and the unemployment rates for the two skill groups are essentially the same. The equilibrium value of $\theta = 8.01$ implies that the steady-state measure of vacancies is quite high, namely, $v = 16.02$. The average duration of unemployment is slightly more than two months $(12 \times (1/5.66) = 2.12),$ while the average duration of a vacancy is a bit less than 1.5 years. (The average duration of a high-skill vacancy is approximately twice that of a low-skill vacancy, but almost all vacancies are low-skill.) Note finally that even though jobs are vacant for extended periods and that the fixed cost, c, is incurred whether the job is occupied or not, once the vacancy is filled, most of the output generated by the match goes to the worker in the form of wages. For example, when a low-skill worker is employed on a low-skill job, a flow output of $s_1 = 1.0$ is generated. Eighty-five percent of this flow output goes to the worker as a wage, 10% is required to cover the fixed cost, so only 5% is left as profit. The fact that employers are squeezed in this way is a result of our assumption that the value of maintaining either type of vacancy must be zero. On average, a low-skill vacancy generates a cost of $c = 0.1$ for a bit less than a year and a half, and this is followed by a profit of about 0.05 for slightly more than three years. Discounting using an interest rate of $r = 0.05$ equalizes these expected flows.

Table 1: Numerical Solution with $m(\theta) = 2\theta^{\frac{1}{2}}$

Assumptions: $s_1 = 1,\ p = .5,\ \beta = .5,\ \delta = .3,\ c = .1,\ r = .05$

b	s_2	θ	m	u	γ	ϕ	w_{11}	w_{21}	w_{22}
0	1.1	8.01	5.66	.050	.501	.995	.850	.851	.901
.1	1.1	7.07	5.32	.054	.509	.961	.852	.855	.905
.2	1.1	6.13	4.95	.059	.519	.924	.854	.860	.910
.3	1.1	5.20	4.56	.066	.529	.882	.856	.865	.915
.1	1.2	7.07	5.32	.066	.598	.652	.833	.884	.984
.2	1.2	6.13	4.95	.073	.608	.624	.835	.890	.990
.3	1.2	5.20	4.56	.081	.618	.592	.838	.895	.995

Table 1 also shows the comparative statics effects of varying b and s_2. In rows 2-4, we successively increase b from its baseline value of 0 to $b = 0.1,\ 0.2$, and 0.3. To understand the effects of increasing unemployment compensation, we can refer back to Figure 1. An increase in b shifts both the $V(s_1) - V(s_2) = 0$ and the $V(s_2) = 0$ curves to the left. That is, when b increases, at each value of γ, the equal-value condition and the condition that the value of a high-skill vacancy be zero both require that labor market conditions move to employers' advantage. An increase in b thus necessarily decreases the equilibrium value of θ, but the effect on γ is indeterminate. Equivalently, one can derive the comparative statics effects of an increase in b by differentiating both sides of equation (19) with respect to b, treating θ as a function of b.

As predicted, when b increases, θ falls. The expected duration of unemployment increases, and the expected duration of a vacancy falls. As a result, it becomes relatively more attractive to open high-skill vacancies; i.e., ϕ falls as b increases. As ϕ declines, there is a corresponding increase in γ; i.e., the average duration of unemployment among low-skill workers rises relative to that among high-skill workers. As a consequence, the value of unemployment for high-skill workers begins to increase relative to the corresponding value for the low-skill unemployed, and this generates more wage dispersion within low-skill jobs. That is, $w_{21} - w_{11}$ increases. Note, however, that the dominant features of the baseline equilibrium continue to prevail; namely, most vacancies are low-skill, and most wage dispersion is between rather than within job types. Again, this follows from our parameterization in which high-skill jobs are only 10% more productive than low-skill jobs.

Rows 5-7 of Table 1 examine the effects of increasing the relative productivity of high-skill jobs. Instead of the 10% productivity gap that we assumed in our baseline parameterization, we now set $s_2 = 1.2$. An increase in s_2 can be interpreted as a result of skill-biased technological change. Note from equation (19) that increasing s_2 has no effect on θ. Nonetheless, widening the productivity gap between high- and low-skill jobs has a dramatic effect. Comparing row 2 ($b = .1$ and $s_2 = 1.1$) with row 5 ($b = .1$ and $s_2 = 1.2$), we see a striking decrease in ϕ (from .961 to .652). Since there are now many vacancies for which the low-skilled are unqualified, the fraction of low-skill workers among the unemployed goes up; specifically, γ increases from .509 to .598. Accordingly, the average duration of unemployment among low-skill workers increases, and the value of unemployment for these workers falls. This in turn implies that the wage of low-skill workers must fall (.852 in row 2 versus .833 in row 5). High-skill workers, on the other hand, are, of course, better off as s_2 increases. As ϕ decreases, unemployed high-skill workers are more likely to match with a high-skill job. Since the average duration of unemployment for these workers is independent of s_2 (because $m(\theta)$ doesn't vary with s_2), the value of unemployment among the high-skilled increases. As a result, the wages of high-skill workers on both low- and high-skill jobs increase. The wage increase for high-skill workers that follows from the increase in s_2 is greater on high-skill jobs. The reason is that on high-skill jobs there is both a direct effect (s_2 increases; i.e., high-skill workers produce more on high-skill jobs) and an indirect effect ($U(s_2)$ increases, so the bargaining position of the high-skilled improves), whereas on low-skill jobs, only the indirect effect operates. Rows 6 and 7 examine the interaction between increasing s_2 and increasing b. Increasing unemployment compensation mostly exacerbates the effects of widening the productivity gap. The only exception is that increasing b protects the low-skilled to some extent from the effects of increasing s_2. Increasing unemployment compensation increases the value of unemployment for low-skill workers; hence the wage of these workers is protected to some extent. On the other hand, increasing b causes ϕ to fall even more (e.g., .592 in row 7 versus .652 in row 5), and the unemployment rate among low-skill workers goes up accordingly.

In sum, our model indicates that skill-biased technological change (an increase in s_2) leads to increased wage inequality both within and between skill groups and to increased unemployment among low-skill workers. It is worth emphasizing that the deleterious effects of skill-biased technological change on the welfare of low-skill workers comes about even though these workers are no less productive in an absolute sense than they were before the change.

We have also examined the effects of changing the fraction of the workforce that is low-skill on the equilibrium. Note from equation (19) that changing the skill composition of the workforce does not affect the steady-state ratio of vacancies to unemployment; i.e., θ is independent of ϕ. Nonetheless changing p has a strong effect. In Table 2, we look at the effects of decreasing p (from the baseline level of $p = 0.5$ to $p = 0.4$) for various combinations of b and s_2. The first row of Table 2 can be compared with the corresponding row of Table 1. Increasing the fraction of workers who are high-skill makes high-skill vacancies more attractive, and as a result ϕ falls (from .995 in row 1 of Table 1 to .796 in row 1 of Table 2). The fall in the fraction of low-skill vacancies leads to a relative increase in the representation of the low-skilled among the unemployed (γ falls from .501 to .453, but γ/p increases as p decreases). Average duration of unemployment among low-skill workers increases, and the wage of low-skill workers falls as a result. On the other hand, high-skill workers are more likely to match with high-skill jobs, so the high-skill unemployment value increases. As a result, the decrease in p causes both w_{21} and w_{22} to rise. The remaining rows of Table 2 examine how the decrease in p interacts with changes in b and s_2. Note that with $s_2 = 1.2$ and $b = .2$ or $.3,$ the condition that $s_1 - c \geq rU(s_2)$ is not satisfied. In these cases, the only equilibrium will be one with *ex post* market segmentation.

Table 2: Numerical Solution with $m(\theta) = 2\theta^{\frac{1}{2}}$

Assumptions: $s_1 = 1,\ p = .4,\ \beta = .5,\ \delta = .3,\ c = .1,\ r = .05$

b	s_2	θ	m	u	γ	ϕ	w_{11}	w_{21}	w_{22}
0	1.1	8.01	5.66	.055	.453	.796	.840	.860	.909
.1	1.1	7.07	5.32	.059	.461	.768	.841	.864	.914
.2	1.1	6.13	4.95	.065	.470	.736	.844	.868	.918
.3	1.1	5.20	4.56	.071	.481	.700	.846	.873	.923
.1	1.2	7.07	5.32	.071	.552	.516	.819	.896	.996
.2	1.2								
.3	1.2								

Our final comparative statics exercise examines the effect of increasing $\beta,$ i.e., the workers' share of the net surplus. As one would expect, an increase in β causes wages to rise but also causes unemployment to increase. The unemployment effect of the increase in β can be read off directly from equation (19). The wage effect indicates that the increased share of the net surplus that workers receive when employed more than offsets any weakening in the workers' bargaining position that could result from the increase in unemployment. Table 3 shows the effects of increasing β to 0.6 (as compared with the baseline level of $\beta = 0.5$) for a range of combinations of b and s_2.

Table 3: Numerical Solution with $m(\theta) = 2\theta^{\frac{1}{2}}$

Assumptions: $s_1 = 1$, $p = .5$, $\beta = .6$, $\delta = .3$, $c = .1$, $r = .05$

b	s_2	θ	m	u	γ	ϕ	w_{11}	w_{21}	w_{22}
.1	1.1	4.70	4.34	.066	.507	.971	.861	.863	.923
.2	1.1	4.08	4.04	.071	.516	.933	.862	.867	.926
.3	1.1	3.46	3.72	.079	.527	.890	.864	.871	.931
.1	1.2	4.70	4.34	.080	.595	.657	.846	.886	1.01
.2	1.2	4.08	4.04	.087	.605	.628	.848	.891	1.01
.3	1.2	3.46	3.72	.097	.616	.595	.850	.895	1.02

The Model with Ex Post Market Segmentation

[to be added]

Preliminary Conclusions

In this paper, we have built a model that highlights the role of skill in the labor market. Our definition of skill is such that low-skill workers can only do low-skill jobs, while high-skill workers can do both low- and high-skill jobs, although they are no more productive than low-skill workers on low-skill jobs. This definition makes it possible to examine how low-skill and high-skill jobs interact, in addition to the usual interaction between low-skill and high-skill workers.

Depending on the underlying parameter configuration, our model has two types of equilibria. We have thus far examined the equilibrium in which it is worthwhile for high-skill workers to match with low-skill vacancies, even though high-skill workers are no better at low-skill jobs than low-skill workers are. We refer to this case as "crowding out" since the participation of high-skill workers in the low-skill market has adverse effects on the labor market outcomes for the low-skill workers. This case generates some striking comparative statics results. Most strikingly, a parameter change that we interpret as skill-biased technological change has the effects of increasing wage dispersion both within and between skill groups (as in Katz and Murphy 1992) and of increasing unemployment among low-skill workers (as in Krugman 1994).

In addition to analyzing the case in which it is not worthwhile for high-skill workers to take low-skill jobs, a number of extensions seem worth pursuing. First, in the simplest versions of our model, we can carry out welfare analysis, using, for example, aggregate output as an objective. Second, it should be possible to examine the effects of more sophisticated policy interventions, e.g., a minimum wage or explicitly financed unemployment compensation, on the equilibrium. Third, we might explore the implications of on-the-job search; that is, we might consider a model in which high-skill workers could continue to search for high-skill jobs while employed on low-skill jobs. These and other possible extensions are, of course, for the future. For the moment, we feel that we have demonstrated the usefulness of a model of this type for understanding some of the major current issues in macro labor economics.

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