Differential Fecundity and Gender Biased Parental Investments *

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Abstract

Parents invest differently in sons and daughters. This paper formally re-examines the application of the Trivers Willard hypothesis to humans. Differential fecundity and assortive matching in marriage are necessary for parents to invest differently in the skills of their sons and daughters. Unlike the Trivers Willard hypothesis, better endowed parents do not necessarily invest relatively more on the survival of their sons relative to worse endowed parents. Parents invest more on the health of the child that they value higher. They do not necessarily invest more on the skills of the child that they value higher. The theory generates endogenous population growth.

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1 Introduction

Parents invest differently in their sons and daughters. Some societies favor boys for some investments, others favor girls, and yet others are gender neutral. The evidence is summarized in Section (2). There is no well accepted economic model for these disparate results.

In an influential paper, Trivers and Willard (1973; hereafter TW) proposed a model of gender biased parental investments based on differential fecundity.[1] Their model applies to humans as follows. First, the evolutionary fitness maximization hypothesis implies that men and women act as if they maximize the discounted expected number of descendents. Second, due to child bearing and menopause, women are fecund for shorter periods of their lives than men. Third, if adults can have multiple spouses, males with a lot of resources may have multiple spouses. In this case, males with few resources may have no spouse. Since fecund females are scarce, rich or poor females are likely to have spouses. Fourth, offsprings of rich parents are more likely to be rich compared with offsprings of poor parents. Therefore poor parents should be biased towards investing in the health of their daughters because poor daughters are more likely to have offsprings compared with poor sons. Since rich sons are likely to have more offsprings than rich daughters, rich parents should be biased toward investing in the health of their sons. "*In species with a long period of parental investment after birth of young, one might expect biases in parental behavior toward offspring of different*

[1] Other applications of differential fecundity include Aiyagari, Greenwood and Güner (1997), Andersson (1994), Bergstrom and Bagnoli (1993), Betzig (1997), Krebs and Davies (1991), Ridley (1993), Siow (1998), Trivers (1972), Willis (1996). While analytically less sophisticated, the non-economics literature is much larger and considers a wider range of hypotheses and environments.

*sex, according to parental condition; parents in better condition would be expected to show a bias toward male offspring (*TW, p. 91)."

There are many tests of the TW hypothesis. The empirical evidence for non-human species are inconclusive (Clutton-Brock (1991)). The evidence for humans are also mixed (Cronk (1991), Hrdy (1987), Sieff (1990), Section (2) in this paper). Specifically, the TW hypothesis is not supported in its conclusion. Richer men are more likely to have multiple spouses (E.g. Becker et. al. (1977), Borgerhoff Mulder (1990), Hamilton and Siow (1998), Wolf and MacDonald (1979)), poor men are more likely to be unmarried (E.g. Korenman and Neumark (1991), Loh (1996), Schoeni (1995)), women are more likely to be married than men (E.g. Haines (1996), United Nations (1992)), and the resources of children are positively correlated with that of their parents (E.g. Deolalikar (1993), Glewwe and Jacoby (1994), Lillard and Willis (1994), Parish and Willis (1993), Solon (1997)). But compared with poorer parents, parents with more resources do not always invest more in the health of their sons relative to their daughters.

This paper formally re-examines the application of the TW hypothesis to humans. Analytically, this work builds on Becker's seminal studies on the family (summarized in Becker (1991)). We also borrow liberally from the research program on the economics of the family (Rosenzweig and Stark (1997)). Bergstrom (1994) used the TW hypothesis and a branching model of the family to explain primogeniture in seventeen century Britain and other stratified societies.

In this model, individuals act to maximize expected discounted streams of dynastic consumption. Since there are different forms of parental investments, we distinguish between health and skill investment. Health investment influences child mortality. Conditional on survival, parents decide how much to invest in the skill of the child. Adults are differentiated by marital status, skill level and fecundity. Fecund women are relatively scarce and they command scarcity rents in the marriage market. Parents take into account the future mating and consumption prospects of their offsprings when they invest in their children.

The main results of the model are:

1. Differential gender investments in health reflect the difference in parental valuations of the gender of their children.
2. Differential gender investments in skills do not necessarily reflect the difference in parental valuations of the gender of their children. Skill investment depends on the difference in valuations between a high versus a low skill outcome for the child.
3. Assortive matching in marriage is necessary for gender biased parental investment in skills.[2]
4. Unlike TW, this model does not generate unambiguous predictions about gender biased parental investments in health and the wealth of the parents.
5. There is endogenous population growth.

We differ from TW because TW essentially assumes parents maximize an expected discounted number of children. Whereas in this model, parents maximize an expected dis-

[2] The role of assortive matching on parental investment is anticipated by Behrman, Pollak and Taubman (1986), Foster (1996) and Laitner (1991).

counted stream of dynastic consumption. Due to differential fecundity and assortive matching, high skilled fecund females are relatively scarce and derive scarcity consumption rents. These consumption rents are valued by parents if they discount dynastic consumption. But they are not valued if parents only value the number of descendents. If parents value dynastic consumption, the consumption rents of high skilled daughters may outweigh the discounted consumption of extra grandchildren that high skilled sons may have. Thus in this model, parents may not favor the survival of sons more as their wealth increase.

Empirically, there is little support for the fitness maximization hypothesis among humans (Eckstein, et. al. (1997), Kaplan, et. al. (1995), Rosenzweig and Stark, Vining (1986)). Beginning with Becker, researchers have investigated broader models where parents value both the quantity and quality of children. The dynastic consumption framework of Barro (1974), Becker and Barro (1988) used here is an analytically convenient parametrization of these broader concerns.

2 Empirical Literature Review[3]

Many empirical studies of gender biased parental investments can be discussed in the following regression framework:

$$\iota_c = \alpha_1 G_c + \alpha_2 P_c + \alpha_3 G_c * P_c + \alpha_4 R_c + \alpha_5 Z_c + u_c \quad (1)$$

ι_c : measure of investment in child c

G_c : 1 if child is male and 0 if child is female

[3] **Strauss and Thomas (1995) provides a general survey of gender biased parental investments. Rosenzweig and Stark also cover other issues in fertility and parental investments.**

P_c : indicators of parental skills and resources

R_c : measures of rate of return to investment

Z_c : other covariates

u_c : error term of regression

ι_c include years of schooling, nutritional intake, anthropometric measures and bequests. Other researchers use complements of ι_c such child mortality and adult consumption goods. Different studies include different regressors and control for different properties in u_c.

We now provide a framework for evaluating the empirical evidence.[4]

Let there be two types of parental investments: survival, i_c and skills, k_c. Parents can invest i_c on the survival of a child at a cost $\gamma(i_c)$. $\gamma(.)$ is increasing and convex. With probability i_c, the child will survive into adulthood. Conditional on surviving, parents can further invest k_c on the skill of the child. With probability $\sigma(k_c, P_c)$, the child will become a high skilled adult and with probability (1-$\sigma(k_c, S_p)$), the child will become a low skilled adult. σ is increasing, concave and $\sigma_{k_c P_c} > 0$.

Assuming separability between parental consumption and child welfare, and constant marginal utility of parental consumption, the parents' investment problem is:

$$\max_{i_c, k_c}\{i_c V(P_c, c) - \gamma(i_c)\}$$

$$\text{s.t. } V(P_c, c) \;=\; \max\{\sigma(k_c, P_c)B_c^h + (1 - \sigma(k_c, P_c))B_c^l - k_c\}$$

[4] We build on Behrman, Pollak and Taubman (1986), Behrman, Rosenzweig and Taubman (1994), Davis and Zhang (1995), Rosenzweig and Schultz (1982), Wolpin (1997).

$$B_c^h \quad : \quad \text{benefit from a high skilled child}$$

$$B_c^l \quad : \quad \text{benefit from a low skilled child}$$

First order conditions for the above maximization problem are:

$$\begin{aligned} V(P_c, c) &= \gamma'(i_c^*) \Rightarrow \frac{\partial i_c^*}{\partial V(P_c, c)} > 0 && (2) \\ \sigma_{k_c}(k_c^*, P_c)\Delta B_c &= 1 \Rightarrow \frac{\partial k_c^*}{\partial \Delta B_c} > 0 && (3) \end{aligned}$$

(2) equates the marginal benefit of investing in survival, the expected value of the surviving child $V(P_c, c)$, to the marginal cost of that investment. Optimal investment in survival is increasing in $V(P_c, c)$. Put another way, parents will invest more on the survival of the child that they value more. If parents value boys (b) more, they will invest more on the survival of boys rather than girls (g). Ceteris paribus, the estimate of α_1 in equation (1), where i_c is the inverse of child mortality, can be interpreted as a measure of parental preferences for sons versus daughters.

Optimal investment in skills is motivated differently. (3) shows that the optimal level of investment in skills is determined by the difference in the benefit from a high skilled child versus a low skilled child. Parents will invest more on the skill of a child if the return to that investment is higher. They will not necessarily invest more on the child that they value more. If parents value girls higher than boys, girls will live longer than boys. But parents may invest more on the skills of boys rather than girls. Thus in general, the estimate of α_1 in equation (1), where ι_c is some measure of skill investment, cannot be interpreted as a measure of parental preferences for sons versus daughters. Rather, depending on other covariates, it provides a partial measure of $(\Delta B_b - \Delta B_g)$, the gender difference in the difference in benefits from a high skilled child versus a low skilled child.[5]

If $\gamma''(i_c)$ is approximately constant, the TW hypothesis asserts that $\Delta B_b > \Delta B_g$, together with $\sigma_{k_c P_c} > 0$, implies $V_{P_c}(P_c, b) > V_{P_c}(P_c, g)$ and therefore $\frac{\partial i_b^*}{\partial P_c} > \frac{\partial i_g^*}{\partial P_c}$. Ceteris paribus, the TW hypothesis implies that α_3 in equation (1), where ι_c is child mortality, should be negative.

Finally, we can write

$$
\begin{aligned}
i_c^* &= H(V(P_c, c)) = h(B_c^l, \Delta B_c, P_c) && (4) \\
k_c^* &= f(\Delta B_c, P_c) && (5)
\end{aligned}
$$

This paper provides a theory of the determinants of P_c, B_c^h and B_c^l.

Most empirical researchers do not consider different motivations for different types of parental investments.[6] As shown in the table below, most studies also ignore the rate of return to parental investments. Rosenzweig and Schultz (1982) were the first to empirically recognize that investment in a child is affected by a market rate of return to that investment. They showed that for rural India, as the employment rate of women (men) increases, infant mortality for girls relative to boys fall (rises). Behrman (1988) and Pitt, Rosenzweig and Hassan (1990) also included measures of own market rates of return to explain parental

[5] This interpretation of α_1 is independent of whether i_c affects σ or not. With constant marginal utility of parental consumption, the optimal level of skill investment is not informative about parental valuation of the child.
[6] They do distinguish between investments and transfers.

investments. Behrman, Pollak and Taubman went further by also including a market rate of return of the spouse of the child in determining parental investment.

Among studies of infant and child mortality, there is weak evidence in favor of the TW hypothesis in a few societies.[7] We found only one study using a nationally representative data set which shows unambiguous support for the TW hypothesis (Gu and Roy (1995)).

Foster showed that parental skills affect the skill acquisition of children. Furthermore, adults take this intergenerational correlation into account when they choose their marriage partners. This paper studies the resulting marriage market equilibrium when such concerns are important.

Unlike studies using direct measures of ι_c, Deaton and his colleagues do not find evidence of gender bias when they examine how the consumption of adults vary with the gender composition of their children (summarized in Chapter 4, Deaton (1997)). We suggest that measures of parental consumption, which are proxies for all child investments and consumption, obscure gender differences across different types of child investments and consumption.

Finally, many studies of parental investments find birth order effects. Gender differences in child mortality in South India show up primarily in worse treatment of higher birth order girls. Garg and Morduch (1997) also find sibling effects in Ghana. The evidence on sibling effects for the US is weaker (Butcher and Case (1994), Kaestner (1997)). Unlike the Philippines, bequests in the US show no sibling or gender effect. While sibling effects are absent in the current framework, they are not inconsistent with it and should be included in further investigations.

[7] Also see Hrdy, Cronk (1991) and Sieff.

Table 1: Studies of Gender Biased Parental Investments

AUTHORS	SAMPLE	ι_c	G_c	$G_c * P_c$	R_c
Johansson, Nygren (1991)	China	SR	b		
Gu, Roy (1995)	China	SR	b	b	
Minturn, Stashak (1982)	57 societies	I	n,b^w		
Gaulin, Robbins (1991)	US	IC		n,b^w	
Cronk (1989)	Mukogodo, Kenya	M,N		b	
Rosenzweig, Schultz (1982)	Rural India	M		g	✓
Muhuri, Preston (1991)	Matlab, Bangladesh	M	b	n,b^w	
Das Gupta (1987)	Rural Punjab, India	M	b	n,b^w	
Dyson, Moore (1983)	India	M	n,b		
Borgerhoff Mulder (1989)	Kipsigis, Kenya	M		n,b^w	
Pebley, Stupp (1987)	Rural Guatemala	M	n	b	
Voland (1984)	Historical Germany	M		b	
Svedberg (1990)	Sub-Saharan Africa	A	n		
Horton (1988)	Bicol, Philippines	A	g^w		
Behrman (1988)	Rural India	A,N	b	b^w,g	✓
Behrman, Deolalikar (1989)	Rural South India	N	n	n,b,g	
Strauss (1990)	Rural Côte d'Ivoire	A	n		
Pitt, Rosenzweig, Hassan (1990)	Rural Bangladesh	N	n		✓
Behrman, Pollak, Taubman (1986)	US	S	g^w		✓

AUTHORS	SAMPLE	ι_c	G_c	$G_c * P_c$	R_c
Foster (1996)	Matlab, Bangladesh	S	b		
Lillard, Willis (1994)	Malaysia	S	n	n,g	
Deolalikar (1993)	Indonesia	S	b	n,g	
Chernichovsky (1985)	Rural Botswana	S	g		
Davis, Zhang (1995)	Philippines	S,B	n,b	n,b	
Menchik (1980)	Connecticut	B	n		
Wihelm (1996)	US	B	n		
Deaton (1997)	6 developing countries	AC	n		

SR: reported sex ratio of male births to female births, I: incidence of infanticide, IC: measures of infant welfare, M: infant and child mortality rate, N: nutritional intake measures, A: anthropometric measures, S: years of schooling, B: bequests, AC: measures of adult consumption, b: parents favor boys, b^w: parents weakly favor boys , g^w: parents weakly favor girls, n: gender neutral, g: parents favor girls

3 Preferences and Technology

The society has one good.

Male and female adults are potentially infinitely lived. Each adult who is alive in the current period, will live in the next period with probability β. For any variable α, $\overline{\alpha} = 1 - \alpha$. Thus $\overline{\beta} = 1 - \beta$ is the death hazard. The discount factor of an individual is $\delta < 1$.

In the first period of their adult lives, all individuals are fecund. Males continue to be fecund for the rest of their lives. Each fecund female has probability μ of remaining fecund in the next period. Infertility is irreversible. Differential fecundity is the only exogenous difference between males and females in this society.

Adults also differ by skill levels. There are two skill levels, high (h) and low (l). An adult with skill level j will have j amount of income to spend in each period of his or her life. The skill levels of a husband and wife pair may also interact in the production of skills of their children. We do not distinguish between investing in market versus non-market skills. Thus high and low skills in this model do not correspond to high and low market skills in a multi-good world.

Adult males and females may marry. Abstracting from all other roles for marriage, adults choose to marry because they want to have children. A marriage can end in two ways. If the wife becomes infertile, the couple divorces because there is no further gain from continuing the marriage. The marriage also ends when a spouse dies. When a marriage ends, each surviving ex-partner suffers a utility loss of λ. A fecund widow or widower may remarry. A divorced man may remarry. Since the only reason to marry is to have children, divorced women will not remarry.

In this paper, we model the value of a child to the parents as the appropriately discounted consumption of the child and descendents of the child.

At the beginning of every period of a marriage, the wife will give birth to a boy and a girl. In general, we denote male variables and choices with lower case letters and female variables and choices with upper case letters. Let the parents invest i units of health on the boy at a cost of $\gamma(i)$. $\gamma(.)$ is an increasing convex function. The boy will survive in the current period with probability i. Let the parents invest I units of health on the girl at a cost of $\gamma(I)$ with corresponding survival probability I. After the health investments, there are four possible survival states:

$$\phi = \begin{cases} 1: & \text{son survives} \\ 2: & \text{daughter survives} \\ 3: & \text{son \& daughter survive} \\ 4: & \text{no survivor} \end{cases}$$

If the boy survives, the parents has to further decide how much human capital, k, to invest in the boy at a cost of k. Let s and S be the skill levels of the father and mother respectively. With probability $\sigma(k,S,s)$, the boy will acquire a high skill level, h, as an adult in the next period (the second period of his life). $\sigma(.)$ is increasing and concave in k. It is also supermodular in k, S and s.[8] With probability $\overline{\sigma(k,S,s)}$, the boy will acquire a low skill level, l, as an adult. If the girl survives, the parents may invest K amount of human capital in her. She will acquire skill level h with probability $\sigma(K,S,s)$ and skill

[8] Supermodularity implies all positive cross partials.

level l with probability $\overline{\sigma(K, S, s)}$. For purely pedagogical convenience, we assume parents cannot transfer resources to their offsprings when their offsprings become adults.[9] Thus all parental investments in an offspring occurs in the first period of the offspring's life. When a child becomes an adult, the child is responsible for his or her own decisions. We ignore the children's consumption.

In every period, fecund parents consume two goods: (i) a public good which is the expected discounted utility of current offsprings, and (ii) private consumption. As emphasized by Bergstrom (1997) and Weiss (1997), it is convenient to use transferable utilities to model the preferences of parents when there are public and private goods within a household. We will also assume that there is no loan market.

The relevant market in this society is the marriage market. Women make marriage offers to men.[10] In order to get a man to accept her offer, a woman must provide him his reservation utility. Men are distinguished by their marital status and skill level. From the point of view of marriageable women, all unmarried men with the same skill level, including widowers, are the same. Women are distinguished by their fecundity, marital status and skill level. Similarly, unmarried men regard all fecund women with the same skill level, including widows, as equivalent.

We will consider stationary marriage market equilibria where the equilibrium reservation utilities achieved by men are constant over time.

The analysis of the model is as follows. First, we consider the behavior of individuals

[9] Due to our transferable utility assumption, the inability to make transfers will have no effect on equlibrium parental utilities, marital choices or investment decisions.

[10] Allocations are unaffected if men make marriage offers due to our transferable utilities assumption.

given equilibrium reservation utilities. Then we will show that an equilibrium with those reservation utilities and the desired properties exist.

It is convenient to begin the analysis with the behavior of single men and women.

4 Singles

Given the stationary environment, if a woman chooses to be single in any period, she will weakly prefer to be single in the next period. Let her have skill level S. In each period that she is alive, she will consume S and attain a current period utility of S. Taking the survival probability and a discount factor of δ into account, her current expected discounted lifetime utility is:

$$W^S = \frac{S}{1 - \delta\beta} \tag{6}$$

When a married woman becomes infertile, her marriage ends and she will not remarry. $W^S - \lambda$ is her lifetime utility as a divorced woman.

Similarly, a single male of skill level s will attain an expected discounted utility of:

$$w^s = \frac{s}{1 - \delta\beta}$$

5 Newly Married Men

Let an unmarried man have skill level s. If a woman wants to marry him, she has to give him an expected discounted utility of r^s which is the reservation utility that he can get by marrying somebody else. Assume that there are married men with high and low skills. Then since men can always choose to remain single,

$$r^s \geq w^s \tag{7}$$

6 Value Functions of Married Women

Let the value function of a fecund woman with skill level S who marries a man of skill level s and has to provide him with a reservation value of r be:

$$V^m(S, s, r)$$

7 Value Function Of A Fecund Marriageable Woman

Let the value function of a fecund marriageable woman with skill level S be V^S. Then

$$V^S = \max\{W^S, V^m(S, h, r^h), V^m(S, l, r^l)\} \tag{8}$$

8 Lifetime Utility of a Married Man

Consider a man with skill level s who is married to a woman of skill level S. The married couple will produce a baby boy and girl in the current period. Let I and i be the respective health investments on the girl and the boy. Depending on the survival state ϕ, let the respective human capital investments on the girl and boy be $K(\phi)$ and $k(\phi)$. In state ϕ, let the consumption of the husband be $g(\phi)$. In the next period, if the marriage remains fecund, let the man receive a continuation value of $v^{'}$. The expected lifetime utility of the married man is:

$$v^m(s) \quad = \quad \mathbf{E}_\phi[g(\phi) + \delta\beta\{\mathbf{E}_t(r^t|k(\phi), \phi, S, s) + \mathbf{E}_T(V^T|K(\phi), \phi, S, s)\}|I, i] \tag{9}$$

$$+\delta\beta\{\beta\mu v^{'} + \overline{\beta\mu}(r^s - \lambda)\} \tag{10}$$

$$= \mathbf{E}_\phi[g(\phi) + \delta\beta\check{E}(\phi)|I, i] + \delta\beta\{\beta\mu v^{'} + \overline{\beta\mu}(r^s - \lambda)\}$$

$$\check{E}(\phi, S, s) \quad : \quad \mathbf{E}_t(r^t|k(\phi), \phi, S, s) + \mathbf{E}_T(V^T|K(\phi), \phi, S, s)$$

$\mathbf{E}_z$ is the expectations operator with respect to the random variable z. If the boy survives and grows up, he will have skill t equal to h or l. $\delta\beta\mathbf{E}_t(r^t|k(\phi), \phi, S, s)$ is the expected value of the son. If the girl survives and grows up, she will have skill T equal to h or l. $\delta\beta\mathbf{E}_T(V^T|K(\phi), \phi, S, s)$ is the expected value of the daughter. So the value of children to parents is modelled as the appropriately discounted consumption flows of a child and his or her descendants. The terms in (10) summarize the expected discounted returns from surviving into the next period. The first term within braces in (10) is the probability that the marriage remains fecund in the next period multiplied by the value of continuation in such a marriage. The second term is the probability of the marriage ending multiplied by the value of the end of the marriage.

9 Decision Problem Of A Married Woman

Consider the decision problem of a fecund woman of skill level S who is married to a man of skill s and has to give him a reservation utility of r. In the current period of marriage, they will have a boy and a girl. Depending on the survival state ϕ, let the consumption of the woman be $G(\phi)$. She must choose $g(\phi), k(\phi), K(\phi), i, I$, and $v^{'}$ such that he gets at least his reservation utility r. That is:

$$v^m(s) = \mathbf{E}_\phi[g(\phi) + \delta\beta\check{E}(\phi, S, s)|I, i] + \delta\beta\{\beta\mu v^{'} + \overline{\mu\beta}(r^s - \lambda)\} \geq r \tag{11}$$

Her budget constraint when state ϕ occurs is:

$$g(\phi) + G(\phi) + k(\phi) + K(\phi) + \gamma(i) + \gamma(I) \leq s + S \tag{12}$$

We assume husbands and wives can enforce their within marriage allocation agreement.[11]

Let $\Omega' = \{g(\phi), G(\phi), k(\phi), K(\phi), i, I, v^{'}\}$. The woman will solve:[12]

$$\begin{aligned} V^m(S, s, r) &= \max_{\Omega'} \mathbf{E}_\phi[G(\phi) + \delta\beta\check{E}(\phi, S, s)|I, i] \\ &\quad + \delta\beta\{\beta\mu V^m(S, s, v^{'}) + \overline{\beta}\mu(V^S - \lambda) + \overline{\mu}(W^S - \lambda))\} \end{aligned} \tag{13}$$

subject to the constraints (11) and (12).[13]

We begin the analysis by observing that in equilibrium, the constraints, (11) and (12), will hold with equality because she can increase her private consumption without reducing his utility if any constraint is slack.

When (11) holds with equality, we can rewrite it as:

$$\begin{aligned} r &= \mathbf{E}_\phi[g(\phi) + \delta\beta\check{E}(\phi, S, s)|I, i] + b(r^s - \lambda) + dv^{'} \\ b &: \delta\beta\overline{\mu\beta} \\ d &: \delta\mu\beta^2 \end{aligned} \tag{14}$$

Solve for $g(\phi)$ in (12) and substitute it into (14) to get:

[11] The case for efficient intrahousehold allocations is made by Browning, et. al. (1994) and Chiappori (1992). A dissenting view is expressed in Udry (1997).

[12] We study the decision problem of the wife subject to her giving her husband his reservation utility. Since the agents have transferable utilities and decisions are efficient, our choice has no substantive consequence.

[13] A sufficient condition for V^m to be finite is $\delta\beta(2 + \mu) < 1$.

$$r = b(r^s - \lambda) + dv^{'} + \mathbf{E}_{\phi}[s + S - G(\phi) - k(\phi) - K(\phi) - \gamma(i) - \gamma(I) + \delta\beta\check{E}(\phi, S, s)|I, i] \quad (15)$$

Let $\Omega = \{k(\phi), K(\phi), i, I, v^{'}\}$. Solve for $\mathbf{E}_{\phi}G(\phi)$ in (15) and substitute it into (13) to get:

$$\begin{aligned} V^m(S, s, r) &= \max_{\Omega}\{\mathbf{E}_{\phi}[2\delta\beta\check{E}(\phi, S, s) - k(\phi) - K(\phi)|I, i] - \gamma(i) - \gamma(I) \quad (16) \\ &\quad + s + S + b(r^s - 2\lambda) + fV^s + (b - f)W^s + d(v^{'} + V^m(S, s, v^{'}))\} - r \\ f &: \delta\beta\overline{\beta}\mu \end{aligned}$$

(16) shows the benefit of the transferable utility assumption. The reservation utility r is additively separable from the rest of the expression in the rhs of (16). That is, the optimal choice of Ω is independent of r. Put another way,

$$V^m(S, s, r) = \Psi(S, s) - r \quad (17)$$

Noticing that (17) implies $v^{'} + V^m(S, s, v^{'}) = \Psi(S, s)$, (16) may be rewritten as:

$$\begin{aligned} V^m(S, s, r) &= \max_{\Omega}\{s + S + b(r^s - 2\lambda) + fV^s + (b - f)W^s + d\Psi(S, s) - r \quad (18) \\ &+ \mathbf{E}_{\phi}[2\delta\beta\check{E}(\phi, S, s) - k(\phi) - K(\phi)|I, i]\} - \gamma(i) - \gamma(I) \end{aligned}$$

Let z^* denote the optimal value of z. Let $\Delta r = (r^S - r^s)$ and $\Delta V = (V^S - V^s)$. Conditional on the survival of the children, the first order conditions with respect to $k(\phi), K(\phi)$ are:

$$\frac{\partial(2\beta\check{E}(\phi,S,s))}{\partial k(\phi)} = 2\delta\beta\sigma_k(k(\phi)^*,S,s)\Delta r = 1 \tag{19}$$

$$\frac{\partial(2\beta\check{E}(\phi,S,s))}{\partial K(\phi)} = 2\delta\beta\sigma_K(K(\phi)^*,S,s)\Delta V = 1 \tag{20}$$

Due to my separability assumptions on the valuations of children and the investment process, and transferable utilities, parents do not link investments on their son with that of their daughter.[14] (19) equates the marginal benefit of increasing human capital investment in the son to the marginal cost of that investment. The marginal benefit depends on the increase in the probability of the son obtaining a high skill level due to a marginal increase in $k(\phi)$ multiplied by the difference in values between a high skilled boy versus a low skilled one. (20) provides a similar condition for optimal human capital investment in the surviving daughter.

Comparing (19) with (20),

$$k(\phi)^* \lesseqgtr K(\phi)^* \Leftrightarrow \Delta r \lesseqgtr \Delta V \tag{21}$$

(21) says that parents will invest more in the human capital of their son than their daughter when the difference in values between a high skilled son versus a low skilled son is larger than the difference in values between a high skilled daughter versus a low skilled daughter. Similarly, parents will invest less in the human capital of their son than their daughter when $\Delta r < \Delta V$.

[14] These assumptions are for pedagogical convenience. But it does mean that we cannot discuss issues of sibling rivalry.

The relative investments in human capital between sons and daughters do not necessarily reflect the difference in the valuations of the gender of their children.

As $\sigma_1(., S, s)$ is increasing in S and s, we can differentiate (19) and (20) and show that parental investments in human capital for children are increasing in parental skills. For example,

$$\frac{\partial k(\phi)^*}{\partial S} = -\frac{\sigma_{kS}(k(\phi)^*, S, s)}{\sigma_{kk}(k(\phi)^*, S, s)} > 0 \tag{22}$$

Without further restrictions, the model has no prediction as to whether $\frac{\partial k(\phi)^*}{\partial S} > \frac{\partial K(\phi)^*}{\partial S}$ or vice versa.

Conditional on survival, let $\widehat{r(S,s)} = 2\delta\beta\mathbf{E}_t(r^t|k(1\cup 3)^*, 1\cup 3, S, s) - k(1\cup 3)^*$ denote the expected net return to a son. Conditional on survival, let $\widehat{V(S,s)} = 2\delta\beta\mathbf{E}_T(V^T|K(2\cup 3)^*, 2\cup 3, S, s) - K(2\cup 3)^*$ denote the expected net return to a daughter. From (18), the first order conditions with respect to i and I are:

$$\widehat{r(S,s)} = \gamma_i(i^*) \tag{23}$$

$$\widehat{V(S,s)} = \gamma_I(I^*) \tag{24}$$

Comparing (23) and (24), parents will invest more on the health of their daughter if the expected net return from her is higher than that from their son and vice versa. Unlike investments in human capital, the levels of investment in health of their children directly reflect the valuations that parents place on their children.

Take the derivative of i^* with respect to S in (23) to get:

$$\frac{\partial i^*}{\partial S} = -\frac{\widehat{r_S(S,s)}}{\gamma_{ii}(i^*)} = -\frac{2\delta\beta\Delta r\sigma_S(k(1\cup 3)^*, S, s)}{\gamma_{ii}(i^*)} > 0 \tag{25}$$

Optimal health investment in a son is increasing in the skill of the mother. By symmetry, it is also increasing in the skill of the father. Likewise, optimal health investment in a daughter is also increasing in parental skills.

Using (22), (25) and assuming γ'' is approximately constant,

$$\frac{\partial i^*}{\partial S} \gtrless \frac{\partial I^*}{\partial S} \Leftrightarrow \Delta r \gtrless \Delta V \tag{26}$$

If $\Delta r > \Delta V$ as assumed by TW, then the TW hypothesis follows. The next issue is to determine whether $\Delta r > \Delta V$.

10 Marriage Market Clearing

We look for a marriage market equilibrium in which the valuations of men and women by skills are constant over time. We also want an equilibrium with assortive matching by skills in marriage.

We will now discuss four conditions for marriage market clearing. First, because women will become infertile, fecund women will be scarce relative to men. So in equilibrium, some men do not marry. Since marriageable men are only differentiated by skills, we expect that some low skilled men will be unmarried. If other low skilled men marry, then low skilled men must be indifferent between marrying and not marrying. That is, the reservation utility of low skilled men in marriage, $r^l = w^l$.

Second, if there are more high skilled marriageable men than low skilled marriageable women, some high skilled men will marry low skilled women. So low and high skilled women must offer high skilled men the same reservation utility.

Third, if low skilled women marry high and low skilled men, these women must be indifferent between high and low skilled men. That is in equilibrium, $V^m(l,l,r^l) = V^m(l,h,r^h)$.

Finally, when there is assortive matching, high skilled women must prefer to marry high skilled men rather than low skilled men. That is, $V^m(h,l,r^l) < V^m(h,h,r^h)$.

To study the above restrictions on market equilibrium, it will be convenient to let:

$$\pi_{Ss} = \mathbf{E}_\phi[2\delta\beta\check{E}(\phi,S,s)^* - k(\phi)^* - K(\phi)^*|I^*,i^*] - \gamma(i^*) - \gamma(I^*) \tag{27}$$

π_{Ss} is the expected net return to parents from having children.

Also observe that v' is not in the objective function (18). That is, the woman is indifferent between all values of v'. Let $v'^* = r$. Then, (18) may be rewritten as:

$$(1-d)V^m(S,s,r) = s + S + b(r^s - 2\lambda) + fV^s + (b-f)W^s + \pi_{Ss} - (1-d)r \tag{28}$$

11 Competing For High Skilled Men

When low and high skilled women marry high skilled men, they must offer these men the same reservation utility. Using (28) and setting $r = r^h$, the utility that a high skilled men obtains from marrying a high skilled woman is:

$$(1-d)r^h = h + h + b(r^h - 2\lambda) + fV^h + (b-f)W^h + \pi_{hh} - (1-d)V^m(h,h,r^h) \tag{29}$$

Similarly, the utility that a high skilled man obtains from marrying a low skilled woman is:

$$(1-d)r^h = l + h + b(r^h - 2\lambda) + fV^l + (b-f)W^l + \pi_{lh} - (1-d)V^m(l,h,r^h) \tag{30}$$

Since women choose to marry high skilled men in equilibrium, it must be the case that $V^m(S,h,r^h) = V^S$. Substitute V^S for $V^m(S,h,r^h)$ and equate (29) to (30) to get:

$$V^h - V^l = \Delta V = \frac{\pi_{hh} - \pi_{hl}}{1-\delta\mu\beta} + \frac{h-l}{1-\delta\beta} \tag{31}$$

12 Indifference Of Low Skilled Women

When low skilled women marry both high and low skilled men, they must get the same utility from both choices, that is:

$$V^m(l,l,r^l) = V^m(l,h,r^h) \tag{32}$$

Using (28) again, rewrite (32) as:

$$r^h - r^l = \Delta r = \frac{h-l}{1-\delta\beta} - \frac{\pi_{ll} - \pi_{lh}}{1-\delta\beta} \tag{33}$$

13 Assortive Matching

Under assortive matching, high skilled women must prefer to marry high skilled men rather than low skilled men. That is, $V^m(h, h, r^h) \geq V^m(h, l, r^l)$. Using (28) again, this inequality implies:

$$\pi_{hh} - \pi_{hl} \geq (1 - b - d)\Delta r - (h - l) \quad (34)$$

Substitute (33) into (34) to get:

$$\pi_{hh} - \pi_{hl} \geq \pi_{lh} - \pi_{ll} \quad (35)$$

Since $\pi_{hl} = \pi_{lh}$ by assumption, (35) is the well known condition for assortive matching in models of marriage.[15] (35) means that π_{Ss} is supermodular (Milgrom and Roberts (1990)). By theorem 4 in Milgrom and Roberts, π_{Ss} is supermodular if $\sigma(., S, s)$ is supermodular. If $\sigma(., S, s)$ is differentiable, $\frac{\partial^2\sigma}{\partial S \partial s} \geq 0$ implies (35).

14 Gender Biased Parental Investments in Skills

From (21), we know that parents will invest relatively more in the human capital of boys rather than girls when $\Delta r > \Delta V$. This section studies the condition under which that inequality holds.

Subtract (31) from (33) to get:

[15] Becker, Lam (1988), Shimer and Smith (1996).

$$\Delta r - \Delta V = \frac{\pi_{lh} - \pi_{ll}}{1 - \delta\beta} - \frac{\pi_{hh} - \pi_{hl}}{1 - \delta\mu\beta} \tag{36}$$

If there is no productive complimentarity, $\sigma(j, S, s) = \sigma(j)$ and $\pi_{Ss} = \pi$ for all skill matches. The rhs of (36) is equal to zero and $\Delta r = \Delta V$. In this case, there is no gender biased parental investment in skills. Differential fecundity, by itself, do not imply that parents prefer to invest more in boys than in girls. It only implies that fecund women are scarce and will collect rents in the marriage market. Without productive complimentarity, the skill of a potential spouse is irrelevant to a person looking for a marriage partner. A high skilled potential spouse captures all the returns to his or her skill in additional own consumption. His or her high skill does not affect other dimensions of the marriage. All children get the same investments and expected outcomes, independent of the skills of their parents. Boys (girls) of high skilled parents have the same expected level of consumption and offsprings as boys (girls) of low skilled parents.

The lhs of (36) will be positive if:

$$(1 + \frac{\delta\beta\overline{\mu}}{1 - \delta\beta})(\pi_{lh} - \pi_{ll}) > \pi_{hh} - \pi_{hl} \tag{37}$$

An interpretation of (37) is as follows. Condition (35) says that productive complimentarity between parental skills in human capital production is necessary for assortive matching. (37) puts a bound on this complimentarity.

Consider the case where there is a modest amount of assortive matching. High skilled adults are more desirable as marriage partners than low skilled adults because children of high skilled parents are more likely to be skilled. There is a positive correlation between the skills of parents and their children. Both high skilled men and women generate consumption rents in marriages. But high skilled men derive further utility from being able to have more expected offsprings. Low skilled men will have less expected offsprings. High and low skilled women have the same expected number of children. Due to differential fecundity and assortive matching, the difference in valuations between a high and low skilled son is larger than the difference in valuations between a high and low skilled daughter. By (36), parents will invest relatively more on the skills of sons.

Why is a bound on assortive matching necessary for parents to favor investing in the skills of boys? If the bound, i.e. (37), fails, high skilled women and high skilled men are very productive together. A high skilled couple will generate a lot of rent relative to a mixed skill couple. Since high skilled women are scarce relative to high skilled men, the high rent from the high skilled pair will accrue to the woman. The high rent will bid up ΔV relative to Δr and make $\Delta r > \Delta V$ less likely to hold. If (37) fails, parents will prefer to invest more in the skills of their girls relative to their boys.

To summarize, we need differential fecundity and productive complimentarity to get gender biased parental investments in skills. Since productive complimentarity in marriage is to be expected, there will be gender biased parental investments. But the theory does not unambiguously predict that parents will invest more in the skills of boys relative to girls. Put another way, even with differential fecundity and assortive matching, parents may choose to invest more in girls than boys. The productive complimentarity must be modest for parents to invest relatively more in the skills of their sons rather than their daughters.

15 Are Daughters Better Than Sons?

Without assortive matching, $\Delta r = \Delta V$. But since fecund women are relatively scarce, $r^l < V^l$, and parents will favor girls over boys. They will invest more on the health of girls than boys. From (26), there will be no correlation between gender bias in health investment and the skills of the parents. Although the Trivers Willard hypothesis does not hold, the absence of a positive correlation between parental skills and offsprings' skills violates a basic assumption of the Trivers Willard hypothesis.

When there is assortive matching and (37) is not satisfied, then $\Delta r \leq \Delta V$ and parents will want to weakly invest more in the human capital of girls than boys. Furthermore, $V^l > r^l$ because all fecund women choose to marry when a low skilled fecund woman can always attain the utility of a low skilled man by remaining single. Thus conditional on survival, the net expected return to a girl, $\widehat{V(S,s)}$ will exceed that of a boy, $\widehat{r(S,s)}$. In this case, parents will invest more in the health and skills of girls relative to boys. Put another way, if parents invest more in the skills of daughters than sons, we also know that parents value their daughters more. Moreover, from (26), we know that higher skilled parents will invest relatively more on the health of their daughters.

In this equilibrium, there are more high skilled women than men. But since women can become infertile, the stock of high skilled marriageable men may still be larger than the stock of fecund high skilled marriageable women.

When there is assortive matching and (37) is satisfied, $\Delta r > \Delta V$ and parents will want to invest more in the human capital of boys than girls. But since $V^l > r^l$, we cannot rank $\widehat{V(S,s)}$

and $\widehat{r(S,s)}$ without further restrictions. Without productive complimentarity, $\Delta r = \Delta V$ and $\widehat{V(S,s)} > \widehat{r(S,s)}$. By continuity, with small amounts of productive complimentarity, $\Delta r > \Delta V$ and parents will invest more in the human capital of their sons. But still $V^l > r^l$ and parents will invest more on the health of their daughters. So if parents invest more in the skills of their sons rather than their daughters, it does not imply that parents value sons more than daughters. In general, we cannot rank the valuations of boys versus girls. In numerical simulations, we have found parameters where $\Delta r > \Delta V$ and $\widehat{V(S,s)} < \widehat{r(S,s)}$. From (26), we know that higher skilled parents will invest relatively more on the health of their sons.

16 The TW Hypothesis Revisited

TW predicts that resource poor parents are more likely than resource rich parents to invest in the survival of their daughters. This model does not give such an unambiguous prediction. The difference between the two models is due to different assumptions about preferences in the two models.

We paraphrase the TW argument as follows. Due to differential fecundity, males with a lot of resources may have multiple mates. Males with few resources may have no mate. Since fecund females are scarce, resource rich or poor females are likely to have mates. Offsprings of resource rich parents are more likely to be rich compared with offsprings of poor parents. The evolutionary fitness hypothesis implies that parents act as if they trade-off between own consumption and a discounted expected number of descendents. Then resource poor parents should at the margin be biased towards investing in the health of their daughters since poor daughters are more likely to have offsprings compared with poor sons. Resource rich parents should at the margin be biased toward investing in the health of their sons since rich sons are likely to have more offsprings than rich daughters.

In this model, resource rich males have more children than rich females.[16] Resource poor females have more children than poor males. Resource rich parents invest more in the human capital of their children and thus are more likely to have rich children. But parents maximize an expected discounted sum of dynastic consumption. In this case, resource rich parents do not necessarily choose to invest relatively more on the health of their sons compared with poor parents.

The difference between discounting dynastic consumption versus discounting the number of descendents is subtle but significant. Due to productive complimentarity and assortive matching, when fecund resource rich females are scarce relative to rich males, fecund resource rich females derive rents from their scarcity. From the point of view of a parent, the extra own consumption of a resource rich female child may outweigh the discounted consumption of the extra descendents of a resource rich male child. When discounted dynastic consumption matters, a resource rich daughter may or may not be more valuable than a resource rich son. This ambiguity is responsible for the ambiguous prediction about the gender bias in health investments as parental wealth increases. If parents only value the total number of children and not their consumption, the extra consumption derived by a resource rich daughter is

[16]Offsprings of high skilled females are likely to have more decendents than offsprings of high skilled males since some high skilled males will marry low skilled females. This effect will mitigate the direct advantage of high skilled males over high skilled females in generating decendents.

irrelevant. Only the discounted number of offsprings matter and resource rich sons will have more offsprings than resource rich daughters.

The different predictions between TW and this model are due to the different assumptions about preferences. Following Becker and the literature on economics of the family, we model parents as facing a trade-off between own consumption and the quantity and quality (as measured by children consumption) of children. TW model parents as facing a trade-off between own consumption and the quantity of children. The quality of children matters only as it affects the number of future descendents (Rogers (1990)). It has no direct effect on parental welfare. The quality of girls matters indirectly because it affects the quality of future boy offsprings.

17 Equilibrium Quantities

Let sS_t be the number of married couples at time t with skills s and S. s_t (S_t) is the number of single men (women) at time t with skill l. Since equilibrium prices are constant over time, survival probabilities and the probabilities of attaining high skills are time independent. φ_{sS} is the equilibrium probability that a couple with skills s and S will have a high skilled son. φ_{sS} incorporates both survival probability and the probability of successful human capital investment. Let ω_{sS} be the equilibrium probability that a couple with skills s and S will have a high skilled daughter. $\overline{\varphi_{sS}}$ $(\overline{\omega_{sS}})$ is the equilibrium probability that a couple with skills s and S will have a low skilled son (daughter). Consider the evolution of the number of high skilled men who all have wives:

$$hh_{t+1} + hl_{t+1} = \beta(hh_t + hl_t) + \varphi_{hh}hh_t + \varphi_{hl}hl_t + \varphi_{ll}ll_t \tag{38}$$

The first term on the rhs of (38) is the number of surviving high skilled men inherited from the previous period. The next three terms consist of the number new skilled men who enter the marriage market.

Consider the evolution of the number of low skilled men with wives and low skilled single men:

$$l_{t+1} + ll_{t+1} = \beta(ll_t + l_t) + \overline{\varphi_{hh}}hh_t + \overline{\varphi_{hl}}hl_t + \overline{\varphi_{ll}}ll_t \tag{39}$$

Consider the evolution of the number of low skilled married women:

$$hl_{t+1} + ll_{t+1} = \mu\beta(hl_t + ll_t) + \overline{\omega_{hh}}hh_t + \overline{\omega_{hl}}hl_t + \overline{\omega_{ll}}ll_t \tag{40}$$

Consider the evolution of the number of high skilled married women:

$$hh_{t+1} = \mu\beta hh_t + \omega_{hh}hh_t + \omega_{hl}hl_t + \omega_{ll}ll_t \tag{41}$$

We do not have to keep track of single women in this model because these women do not affect reproduction.

Use (41) and (38) to get:

$$hl_{t+1} = (\varphi_{hh} - \omega_{hh} + \beta\overline{\mu})hh_t + (\varphi_{hl} - \omega_{hl} + \beta)hl_t + (\varphi_{ll} - \omega_{ll})ll_t \tag{42}$$

Combine (42) and (40) to get:

$$ll_{t+1} = (\overline{\omega_{hh}} + \omega_{hh} - \varphi_{hh} - \beta\overline{\mu})hh_t + (\overline{\omega_{hl}} + \omega_{hl} - \varphi_{hl} - \beta\overline{\mu})hl_t \tag{43}$$
$$+(\overline{\omega_{ll}} + \omega_{ll} - \varphi_{ll} + \mu\beta)ll_t$$

Combine (39) and (43) to get:

$$l_{t+1} = (\overline{\varphi_{hh}} + \varphi_{hh} - \omega_{hh} - \overline{\omega_{hh}} + \beta\overline{\mu})hh_t + (\overline{\varphi_{hl}} + \varphi_{hl} - \omega_{hl} - \overline{\omega_{hl}} + \beta\overline{\mu})hl_t \tag{44}$$
$$+(\overline{\varphi_{ll}} + \varphi_{ll} - \omega_{ll} - \overline{\omega_{ll}} + \beta\overline{\mu})ll_t + \beta l_t$$

We assume that, within a family,

$$\beta\overline{\mu} > (\omega_{sS} + \overline{\omega_{sS}}) - (\overline{\varphi_{sS}} + \varphi_{sS}) \tag{45}$$

the difference in the probabilities of survival between daughters and sons is smaller than the re-entry rate of fathers into the marriage market due to the mother becoming infertile, $\beta\overline{\mu}$. Then the coefficients in the difference equation (44) are all positive. As long as the number of high skilled males, high and low females are positive, the number of single low skilled males will be positive. Moreover l_t does not affect hh_{t+1}, hl_{t+1} nor ll_{t+1}. So we may ignore the number of single males, i.e. equation (44), in studying the evolution of equilibrium quantities in this model.

Let $X_t = [hh_t, hl_t, ll_t]'$. The three equation system, (41) to (43) can be written compactly as:

$$X_{t+1} = AX_t \tag{46}$$

18 Solutions

We look for solutions to (46) where the equilibrium quantities of each type of marriages grow at the same constant rate. That is, a solution exists if there is a strictly positive eigenvalue, λ, and strictly positive eigenvector, $X = [hh, hl, ll]'$, of A such that for some $c > 0$,

$$\begin{aligned} \lambda X &= AX \\ X_t &= c\lambda^t X \end{aligned}$$

If a solution exists, λ is equal to one plus the growth rate of the population. For the society as a whole, the long run growth rate of marriages must be equal to and cannot exceed the long run growth rate of fecund women in the society. Thus if a solution exists,

$$\lambda = \mu\beta + \frac{(\overline{\omega_{hh}} + \omega_{hh})hh + (\overline{\omega_{hl}} + \omega_{hl})hl + (\overline{\omega_{ll}} + \omega_{ll})ll}{hh + hl + ll} \tag{47}$$

The restrictions on A implied by the theory are:

$$\begin{aligned} \omega_{hh} &\geq \omega_{hl} \geq \omega_{ll} \\ \overline{\varphi_{hh}} + \varphi_{hh} &\geq \overline{\varphi_{hl}} + \varphi_{hl} \geq \overline{\varphi_{ll}} + \varphi_{ll} \\ \overline{\omega_{hh}} + \omega_{hh} &\geq \overline{\omega_{hl}} + \omega_{hl} \geq \overline{\omega_{ll}} + \omega_{ll} \end{aligned}$$

We consider three cases.[17]

[17]There is no general proof of existence in two sex marriage models (Pollak (1990)). Harpending and Rogers (1990) and Rogers (1995) numerically simulate related models.

18.1 A is non-negative

This case comes closest to the TW hypothesis but is not implied by it.

The first row of A is non-negative.

A sufficient condition for the second row of A to be non-negative is when parents invest sufficiently more on the skills of sons such that

$$\varphi_{sS} > \omega_{sS} \tag{48}$$

The third row is non-negative when

$$\overline{\omega_{sS}} + \omega_{sS} > \varphi_{sS} + \beta\overline{\mu} \quad , sS = hh, hl \tag{49}$$

(49) says that the probability that the family with a high skilled father produces a surviving daughter should exceed its probability of producing a high skilled son and the father returning to the marriage market due to the mother becoming infertile.

When (48) and (49) hold, A is non-negative. In this case, the Perron Frobenius theorem for irreducible non-negative square matrix states that there is always one positive eigenvalue, λ, which is larger in absolute value than all other eigenvalues, and whose associated right eigenvector, X, has positive elements. Thus there is at least one solution to (46) in which all types of marriages grow at the same rate.

18.2 A has negative elements

When parents produce more high skilled daughters than sons, $\varphi_{sS} < \omega_{sS}$. But since women will become infertile, the stock of high skilled men may still exceed the stock of high skilled fecund women and the arbitrage conditions can remain. In this case, $(\varphi_{ll} - \omega_{ll})$ in A is negative and the above results do not apply.

Consider the case where the survival rates of girls from all types of families are similar,

$$\omega_{sS} + \overline{\omega_{sS}} \approx k \tag{50}$$

In this case, when a solution exists, $\lambda = (k + \beta\mu)$. Normalize the first element of the eigenvector X, $hh = 1$. Then we can solve $(k + \beta\mu)X = AX$ to get:

$$hl = \frac{\overline{\omega_{hh}}\varphi_{ll} - \omega_{ll}(k - \varphi_{hh} - \beta\overline{\mu})}{\omega_{hl}\varphi_{ll} + \omega_{ll}(k - \varphi_{hl} - \beta\overline{\mu})}$$

$$ll = \frac{\overline{\omega_{hh}}(k - \varphi_{hl} - \beta\overline{\mu}) + \omega_{hl}\omega_{ll}(k - \varphi_{hh} - \beta\overline{\mu})}{\omega_{hl}\varphi_{ll} + \omega_{ll}(k - \varphi_{hl} - \beta\overline{\mu})}$$

Sufficient conditions for hl and ll to be positive are (49) as discussed previously and:

$$\overline{\omega_{hh}}\varphi_{ll} - \omega_{ll}(k - \varphi_{hh} - \beta\overline{\mu}) > 0 \tag{51}$$

The interesting case is when $\varphi_{sS} < \omega_{sS}$. Otherwise (49) and (48) imply A is non-negative and we do not have to impose (51) directly. But when $\varphi_{sS} < \omega_{sS}$, that is when parents produce more high skilled daughters than sons, $(\varphi_{ll} - \omega_{ll})$ in A is negative. But since women will become infertile, the stock of high skilled men may still exceed the stock of high skilled fecund women and the arbitrage conditions remain. If, for all three family types,

$$\frac{\omega_{ll}\beta\overline{\mu}}{\overline{\omega_{hh}} + \omega_{ll}} \geq \omega_{sS} - \varphi_{sS} \geq 0 \tag{52}$$

(51) also obtains. The bound in (52) is proportional to $\overline{\mu}$, the arrival rate of menopause. If $\overline{\mu}$ is large, $\omega_{sS} - \varphi_{sS}$ may be large and there will still be a solution for the reason given just

above. When (50), (49) and (51) hold, we have also found a solution to (46) where each type of marriage grows at the same rate. Note that the solution is exact when child mortality is zero and $k = 1$.

18.3 The general case

When A may have negative elements and the survival rates of girls vary by family type, we have been unable to find analytic restrictions on A to get a solution to (46) which has a constant growth rate. Our conjecture is that such a constant growth rate solution exists.

When $\omega_{sS} - \varphi_{sS} > 0$, we can show that when (49),

$$\min_{sS} \frac{\varphi_{sS} - \omega_{sS} + \beta\overline{\mu}}{\omega_{ll} - \varphi_{ll}} = \alpha_1 \geq \alpha_2 = \max_{sS}\{\frac{\overline{\omega_{sS}} + \omega_{sS} - \varphi_{sS} - \beta\overline{\mu}}{\varphi_{sS}}\} \tag{53}$$

and for some t:

$$hh_t > 0 \tag{54}$$

$$hl_t > 0 \tag{55}$$

$$\alpha_2(hh_t + hl_t) > ll_t > 0 \tag{56}$$

Then, $\forall\ j > 0$,

$$\begin{aligned} hh_{t+j} &> 0 \\ hl_{t+j} &> 0 \\ \alpha_2(hh_{t+j} + hl_{t+j}) &> ll_{t+j} > 0 \end{aligned} \tag{57}$$

That is, given (49), (53) and (54) to (56), all three types of families will not become extinct in finite time. (53) is close to (52) but is not implied by it.

In order to show (57), use (42) to get:

$$hl_{t+1} = \beta\mu hl_t + (\varphi_{hh} - \omega_{hh} + \beta\overline{\mu})hh_t + (\varphi_{hl} - \omega_{hl} + \beta\overline{\mu})hl_t + (\varphi_{ll} - \omega_{ll})ll_t \tag{58}$$

Using (53), (58) becomes:

$$hl_{t+1} \geq \beta\mu hl_t + (\omega_{ll} - \varphi_{ll})(\alpha_1(hh_t + hl_t) - ll_t) \tag{59}$$

Using (53) and (56), the second term on the right hand side of (59) is positive and therefore $hl_{t+1} > 0$. $ll_{t+1} > 0$ follows from (49). $hh_{t+1} > 0$ follows from (54) to (56) and (41).

Using (43) and (53),

$$\begin{aligned} ll_{t+1} &\leq \alpha_2\varphi_{hh}hh_t + \alpha_2\varphi_{hl}hl_t + \alpha_2\varphi_{ll}ll_t + \beta ll_t \\ &\leq \alpha_2(\varphi_{hh}hh_t + \alpha_2\varphi_{hl}hl_t + \varphi_{ll}ll_t + \beta(hh_t + hl_t)) \\ &= \alpha_2(hl_{t+1} + hh_{t+1}) \end{aligned}$$

By induction, (57) follows.

19 Conclusion

This paper presented a formal reconsideration of the TW hypothesis as it applies to humans. Instead of recapitulating the main results from the introduction, we provide some directions for future research.

An extension is to allow for concave utility functions where the marginal utility of consumption is not constant. This generalization will naturally bring in sibling effects. It will also be interesting to see if it is necessary to impose productive complimentarity in investment to get assortive matching in this case.

Another extension is to investigate other assumptions which make equilibrium in this class of two sex marriage models easier to obtain. For example, let the income of each type of worker depend on the total supply of that type in the population.

Siow has a model of differential fecundity with adult own investments in market and non-market skills but there was no parental investment. Another extension is to allow for parental and adult own investments in market and non-market skills (Echevarria and Merlo (1997)).

Aside from the studies cited in footnote (1), economists have ignored the effects of differential fecundity and marriage market considerations on gender roles, welfare consequences of family laws, household formation, and intra-household allocations. The larger agenda is wide open (E.g. Edlund (1998a,1998b)).

Finally, the efficacy of the framework presented here for empirical research on gender biased parental investments remains to be investigated.

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